INSTITUT DES HAUTES ÉTUDES

POUR LE DÉVELOPPEMENT DE LA CULTURE, DE LA SCIENCE ET DE LA TECHNOLOGIE EN BULGARIE

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Concours Général de Physique "Minko Balkanski"

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The main problem and the three exercises are completely independent and can be solved in any order.

All the answers must be given in English or French. The clarity and precision of expression will be taken into account in the attribution of the final note.

The exam is 4 hours long! The calculators are authorized.

Physical constants:

Mass of the Earth: .... $M_E = 6,0.10^{24} kg$
Mass of the Sun: .... $M_S = 2,0.10^{30} kg$
Average Earth radius: .... $R_E = 6,4.10^{6} m$
Average Sun radius: .... $R_S = 7,0.10^{8} m$
Average radius of the Earth's orbit around the Sun: .... $L = 1,5.10^{11} m$
Time needed for one rotation of the Sun around its axe: .... $\theta_S = 2,6.10^{6} s$
Universal gravitational constant: .... $G = 6,67.10^{-11} N.m^2.kg^{-2}$
Perfect gaz constant: .... $\Re = 8,31 J.K^{-1}.mol^{-1}$
Avogadro's number: .... $Na = 6,02.10^{23} mol^{-1}$
Hydrogen molar mass: .... $M_H = 1,0.10^{-3} kg.mol^{-1}$
Speed of light: .... $c = 3,0.10^{8} m.s^2$

Part I

Problem

1 Gravitational Field (8pts)

In this part the Sun and the Earth are supposed to be homogeneous spheres. Using an electrostatic analogy, we are going to find the gravitational field, the gravitational potential and the gravitational energies.

1. Find the electrical field $\vec{E}(A)$, in a point A, created by a point charge q fixed at point O, using: $OA = r$ and $\vec{u}_r = \frac{\overrightarrow{OA}}{r}$.

2. We consider a sperical surface S, centered in O.

   (a) Cite Gauss' theorem for the electrical field.

   (b) Dtermine the flow Φ of the vector $\vec{E}$ going out of S. Note Q_{int} the total charge inside the spherical surface S.

3. We have an uniformely charged ball of radius R, with a total charge Q.

   (a) Determine the electrical field $\vec{E}(r)$ created by this ball at a distance r from its center O for r between 0 and ∞, as a function of Q, $\vec{u}_r$, r and R. Consider two cases $0 < r < R$ and $R < r < +\infty$.

4. (a) Provide the formula of the relationship between the electrostatic energy interaction of the ball in question $(3.a)$ and a point charge q put at a distance r from its center O, and the electrostatic potential $U(r)$ of the ball, found in the previous question.

   (b) Find this energy E_1 (supposed zero in infinity).

5. (a) Remind the expression of the volume density of the electrostatic energy in a point M where the field is $\vec{E}(M)$.

   (b) Show that the electrostatic energy of the ball can be written as $E_2 = \frac{kQ^2}{4\pi\epsilon_0 R}$.

6. Now you can use the formal analogy between the electrical and gravitational field:

   (a) Give the expression of the gravitational field $\vec{g}(A)$, in a point A created by a point mass m fixed in O, using: $OA = r$ and $\vec{u}_r = \frac{\overrightarrow{OA}}{r}$.

(b) Write explicitly the analogous couples (example: charge and mass).

(c) Consider a homogenious ball with radius R and mass M. Determine the gravitational field $\vec{g}(r)$ that this ball creates at a distance r from its center O, for r between 0 and $+\infty$

(d) Numerical application: calculate the intensity g_{SE} of the gravitational field created by the Sun on Earth's surface and also the intensity g_{ES} of the gravitational field created by the Earth on the Sun's surface.

(e) Determine the gravitational energy of interaction between the Earth, assumed as a point, and the Sun.

(f) Numerical application.

7. Find the proper energy of the Sun and the Earth noted respectively, E_{2S} and E_{2T}.

8. Numerical application: calculate these energies and find the energy E_{2TS} of the system Earth-Sun neglecting (in the interaction energy) their radius compared to the distance between the planets. Comment your result.

2 Stability of a spherical star (8pts)

1. Thermal stability.

We are going to use a model of a sperical homogenious star with radius R and mass M made by hydrogen atoms with molar mass M_H, making the hypothesis of a perfect gaz in equilibrium with temperature T uiformely distributed. Every atom has a kinetic energy $e_c = \frac{3}{2}kT$, where $k = \frac{\Re}{Na}$.

(a) Supposing that the Sun is an isolated system, give its total energy E_S.

(b) What is the condition for having a constant radius of the Sun, ie. the Sun is not in infinite expansion?

(c) Give the maximum temperature T corresponding to the previous condition.

(d) Numerical application: calculate this temperature.

2. Dynamical stability.

We are going to use a model of a sperical homogenuous star with radius R and mass M, rotating with a constant period aroud one of its diameters and made by a gaz of particules moving with the same angular speed as the star.

(a) Give the speed of liberation v_i at the surface of the star.

(b) Write a condition for the dynamical stability of the star.

(c) Numerical application: does the Sun verify this dynamical stability? Justify your answer with numerical values.

(d) What happens to a star which has a liberation speed greater than the speed of light? What is the condition for the fraction $\frac{M}{R}$ for these objects? Give the same condition using the density of the star.

(e) Numerical application: calculate for a star with radius R_S, the minimal density that gives a liberation speed greater than the speed of light. Comment this numerical value.

3. Hydrostatic aspects.

   We are going to use a model of a sperical homogenuous star with radius R and mass M, without rotation, whose density $\rho(r)$, internal pressure $P(r)$ and temperature $T(r)$ depend only of the distance r from the center of the star. Furthermore, the star is modelled by perfect gaz of hydrogen atoms in hydrostatic equlibrium, ie. every element of volume dV is equilibrated by the gravitational and pressure forces.

   (a) Remind the fondamental principe of the fluid statics, writing $\overrightarrow{grad}P$ as a function of $\rho(r)$ and $\vec{g}$.

   (b) Give $\frac{dP}{dr}$ in the case of spherical star as a function of G, r, $\rho(r)$ and $M(r)$, the mass contained in the ball with radius r.

   (c) We want to estimate the temperature T_C in the center of the Sun. In order to do that, we suppose that the function $P(r)$ is linear and zero at the surface of the Sun. We will substitute $\rho(r)$ by the average density of the Sun and the term $\frac{M(r)}{r^2}$ by the "average" value $\frac{M_S/2}{(R/2)^2}$. Give P_C and then find T_C, as a function of G, M_S, R_S, $\Re$ and M_H.

   (d) Numerical application: calculate T_C.

Part II

Exercises (4pts)

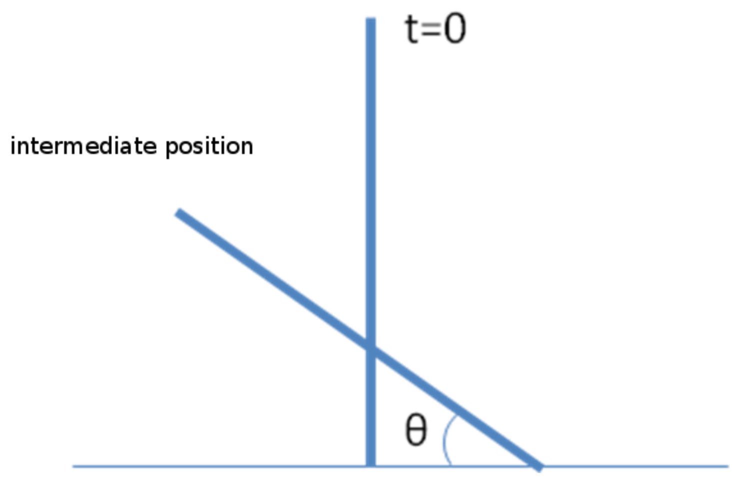
1. A fireworks rocket explodes in the night sky at a height h in an isotrope manner.

   (a) Show that the consisting parts of the rocket form a sphere in every instant of their falling.

   (b) Calculate the radius of the sphere created by these particles when they are all on the ground.

2. A sattelite is on a circular orbit of radius r_0 around the Earth. Show that under some critical value of r_0, the sattelite is being disintegrated ("Roche limit").

3. A vertical ladder falls down without initial speed. The contact with the ground is frictionless.



   (a) Find the speed V_G (of the center of masses) as a function of Ω (the angular speed) and θ. Calculate the speed V_G when the ladder touches the ground entirely.

   (b) Write down the equation of mouvement of the center of masses in the very first moment of the falling. We will note $\alpha = \pi/2 - \theta$. Show that the angle α is inreasing under an exponential law with a caracteristic time $\tau = \sqrt{\frac{L}{6g}}$.